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Productive Thinking: The Role of Perception and Perceiving Opposition[1]

Introduction

The process involved in finding a solution to a problem is defined as "problem solving" (Duncker 1945; Newell & Simon 1972). Recent research on problem solving has emphasized the connection between reasoning and various cognitive abilities (for a review, see Mercier & Sperber 2011) such as intelligence (Burns, Nettlebeck & McPherson 2009), intellect (DeYoung, Braver, Shamosh, Green & Gray, 2009), attention (Unsworth, Spillers & Brewer 2010) and working memory (Unsworth, Spillers & Brewer 2010). Related issues concern the relationship between beliefs and reasoning (Evans, Handley & Bacon 2009), the strength of explanations and evidence in generating and evaluating arguments (Brem & Rips 2000), the role of fast, automatic, unconscious reasoning processes versus processes which are slow, conscious and effortful (Evans 2008). There is also the issue of the impact of the group in modifying individual cognitive biases (Laughlin, Hatch, Silver & Boh 2006).

In this general framework what often remains neglected is the role of perceptual factors. This has occurred despite the importance of perception and visualization which has been clearly shown in studies on the abilities of *processing and manipulating figural features* in geometrical problem solving (Duval 2006; Gutierrez 1996) and the importance of *representational changes* in insight processes (Luo, Niki & Knoblich 2006). The role of perceptual factors in problem solving was first brought up by Gestalt psychologists (Duncker 1926, 1935; Köhler 1920, Maier 1930, 1931a, 1931b, 1945; Wertheimer 1919/1945). Before putting forward our proposal that we hope will stimulate a reconsideration of the role of perception in problem solving and provide initial data supporting it, we will briefly revise how

[1] This article is the paper version of a lecture held at the 18th GTA Scientific Convention, April 11th-14th, 2013, Karlsruhe, Germany.

GESTALT THEORY
 (ISSN 0170-057 X)
Vol. 37, No.1, 7-24

the role of perception in reasoning is dealt with in the three domains of research mentioned earlier.

The Importance of Perception in Productive Thinking

Gestalt psychologists were the first to provide a phenomenological description of what happens when people face a problematical situation (on this, see Nerney 1979). They identified two processes: reproductive thinking (consisting of a mechanical application of chains of associations which have already been learned and reinforced by experience and habits) and productive thinking (a process involving the creation of something new).

Wertheimer (1919/1945) explained that productive thinking starts from a deep understanding of the phenomenal structure of a problem. This structure will suggest the solution:

> "…one of the essential characteristics of a good and genuine solution is that it fits the inner or intrinsic requirements of the problem and is guided by the direction of the task. The inner requirements of problems are as vectors that originate in the problem and the direction of the problem that is taken into account in the process of searching for a correct solution is determined by the directions of these vectors. The relation between a problem and its good solution is the same as the link that joins a question and its answer: a question intends answers that lay in a certain realm. In this way the question could be said to have an orientation or intended direction. If the answer given to it also lays in this direction, then the answer might close the gap opened up by the question. But if it lays in a seemingly unexpected direction, then it is felt to be inappropriate and not to fit the requirements of a question, as happens when a solution does not fit the requirements or direction of a problem in a problem situation" (Luchins & Luchins 1970, pp. 85-86).

Therefore, understanding the structure of a problem by detecting its primary and secondary elements and the relationships between them is essential because the structure not only organizes the problem itself, but also contains gaps or "trouble zones" to be healed which function as cues for the directions to be followed when seeking the solution. It has to be clarified that for Wertheimer the process of understanding the structure of a problem is not a cognitive but a perceptual process. *Seeing* the phenomenal structure of a problem means also *seeing* its solution, that is, the operations to be carried out in order to resolve it. According to Wertheimer (1919/1945), this process consists of the reorganization of the elements of a problem, achieved by dividing what appears to be a unit and unifying what appears to be separate.

The importance of using the phenomenal structure of problems in order to find their solution was also recognized by other Gestalt psychologists who pointed out that the initially perceived structure of a problem may be a potential obstacle to its solution (Duncker 1926, 1935; Harrower 1932; Köhler 1969; Luchins 1942; Maier 1930). Functional fixedness is an example of the type of hurdle which must be overcome. This occurs when objects which have a specific function in everyday life need to be used in a completely different way (Duncker 1935). This is the case with the box of thumbtacks in Duncker's candle problem (Duncker 1945) – the box has to be seen as a rigid horizontal base on which the lit candle can be fixed, and not merely as a container for the thumbtacks. Similarly, Harrower (1932) and Köhler (1969) suggested that the structure of a problem can contrast the discovery of new relationships between the elements of the problem which could be fundamental to the solution. This is evident in, for example, Harrower's duck problem: "swimming under a bridge there came two ducks in front of two ducks, two ducks behind two ducks, and two ducks in the middle. How many ducks were there in all?" (Harrower 1932, p. 113). The solution is: four ducks swimming in single file. But most people will spontaneously organize the two ducks horizontally side by side and the repetition of "two ducks… two ducks… and two ducks" leads them to imagine three horizontal groups each with two ducks swimming side by side therefore believing that the solution must be 6 ducks. If one remains "captured" in this horizontal organization and does not realize that a pair can also be organized vertically (one duck in front of the other) the solution cannot be discovered.

Maier (1930) and Luchins (1942) showed that fixedness is often caused by experience, intended as the habitual way of looking at things and the tendency to apply a procedure, already repeated in a series of similar tasks, to subsequent problems which conversely possess a more direct solution (*Einstellung* effect). This was demonstrated by means of an experiment involving a series of volume-measuring problems, in which three empty containers of different capacity (labeled a, b, c) and a supply of water were provided to participants. They were asked to obtain a stipulated volume of fluid using the containers as tools to measure the desired quantity of fluid. The solution could be reached by applying the formula "b – a – 2c", that is, pouring the water into container "b", taking away from this the quantity of fluid contained in container "a" and subtracting from the remaining water double the quantity of fluid corresponding to the capacity of container "c". In some cases, however, the correct solution could also be reached using a simpler, more direct procedure, i.e. taking away the volume of fluid corresponding to the capacity of container "c" from the quantity of water contained in container "a" (a – c). But the participants kept using the general formula (b – a – 2c), manifesting the *Einstellung* effect.

These authors all suggested that a way to overcome hindrances such as these consists of re-focusing on the phenomenal structure of the problem in order to discover other paths towards the solution.

Learning Geometry: The Importance of Visualization

The importance of figural aspects has been recently emphasized in studies on how we learn geometry. In these studies, geometry is conceived on the one hand as the study of spatial objects, relationships, and transformations that have been formalized (or mathematized), and on the other hand as the axiomatic mathematical systems that have been constructed to represent them (Clements & Battista 1992). In both cases, the processes of construction and visualization of figures are essential and the ability to process and manipulate the visual aspect of figures is considered a relevant cognitive ability. Many studies dealing with learning geometry have focused on the process of visualization, that is, the ability to represent, transform, generate, communicate, document, and reflect on visual information by means of mental images (Duval 2006; Gutiérrez 1996). Visualization is central to thinking about and defining geometrical figures and, in general, space. Geometrical figures are conceived as figural concepts in which conceptual aspects related to the definition of geometrical properties are connected to visual spatial aspects (Baccaglini-Frank, Mariotti & Antonini 2009). Visualization supports all the transformations of figures that can be applied in order to change, explore and manipulate an initial geometrical shape. Duval (2006) distinguished three basic kinds of visual operations that can be performed on figures: mereological transformations, where the figure is divided into new parts or sub-parts that can also be recombined to form a new figure; optical transformations, where a shape is made larger or narrower as if one were using a lens or had located the figure in a 3D space; and transformations of place, where the orientation of the figure in the picture plane with respect to the observer is involved (affecting mainly the recognition of the width of the angles). Of course, when mentally or graphically manipulating geometrical figures by means of these transformations, some properties have to remain invariant by definition (Baccaglini-Frank, Mariotti & Antonini 2009) and this is not easy to learn.

Two spatial abilities which are involved in these mental or figural operations (Gardner 1983) are the ability to manipulate spatial orientation, that is, to understand and then operate on the relationship between the positions of objects in space with respect to one's own position, and the ability to visualize spatially, that is, the comprehension and performance of imagined movements of objects in two- or three-dimensional space (Harris 1981). All this is obviously relevant to problem solving when applied to geometrical tasks and, in general, to any task involving spatial structures. The capacity to see and draw figures using different planes of spatial representation in order to discover the fundamental properties of a given geometrical shape is extremely relevant (Gutiérrez 1996; Gutiérrez, Pegg & Lawrie 2004). Similarly, the difficulty of taking into account different planes of representation when geometrical problems involve two- or three-dimensional space is documented in literature on the subject (Unal, Jakubowski & Corey 2009).

The importance of manipulating different planes of representation is well demonstrated in Gutiérrez et al.'s example (2004): if the task is to draw all the diagonals of a prism such as that shown in Figure 1, various different planes of representation (corresponding to the facets that compose the prism) have to be taken into account in order to reach the correct solution without omitting any of the diagonals.

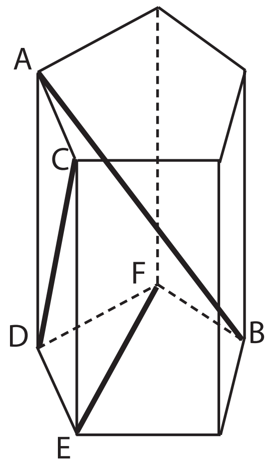

Fig. 1 The figure shows a prism with some of its diagonals. To draw all diagonals people have to consider various different planes of representation, corresponding to the facets composing the prism (The image is drawn by Gutiérrez, Pegg & Lawrie 2004, p. 514).

In conclusion, studies on how we learn geometry demonstrate the importance of visualization. This might seem to be in agreement with the Gestalt psychologists who suggested paying attention to the perceptual structure of a problem. However, in most of the studies on learning geometry, the operations involved in the process of visualization are not conceived as taking place at a perceptual level, but at a slightly superior level – and this is a point of difference with respect to the Gestalt position discussed earlier. According to Duval (1999, p. 14-15) "it [visualization] displays organization of relations, but it is not primitive, because it is not mere visual perception".

A Possible Connection Between Logical and Perceptual Reasoning

We have spoken about the role of the perceptual structure of a problem as a starting point for visualization and mental manipulation of an image. Are these considerations specifically related to geometrical problems? Or, as suggested by Gestalt psychologists, do they apply to reasoning in general, and therefore to logical reasoning as well? We will now take a look at a recent perspective on reasoning which has been developed by Mercier & Sperber (2011) and show how this view can be integrated with the Gestalt psychologists' idea that perceptual structure is essential. In the next section we will go a step further and suggest that a systematic transformation of the characteristics of a problem into their opposites (a process "driven" by the structure of the problem) is a useful strategy in problem solving.

Mercier & Sperber (2011) deal with the kind of reasoning which is involved in tasks where an inference has to be produced. They focus on how this process develops and discuss the errors to which it is exposed. These "informal reasoning fallacies" (Hahn & Oaksford 2007) characterize all activities in which reasoning is implied and therefore also apply to problem solving. When human beings have to solve a problem alone they build a mental model that is composed of premises and conclusions derived from their initial intuitions. Due to a confirmation bias consisting of seeking or interpreting evidence in ways that conform to pre-existing beliefs, expectations, or to a hypothesis they already have, problem solvers seek elements that *support their point of view* without taking into account alternatives. When a group of people as opposed to an individual is involved, this bias is less effective. The presence of other members with different points of view stimulates the consideration of various other facets of the problem and leads them to examine and check the alternative solutions proposed thus correcting the initial bias. According to Mercier & Sperber (2011), this phenomenon, known as the "assembly bonus effect" (Laughlin, Hatch, Silver & Boh 2006), depends on the evolutionary function of reasoning, i.e. the fact that it is based on argumentation. Therefore, it is no surprise that reasoning produces its best results in an argumentative context and particularly in a group, due to the dependence of groups on human communication. In order to achieve a performance similar to that of a group, individual problem solvers have to remove their confirmation bias and make an effort to falsify rather than confirm their initial intuition.

But what type of strategy will contrast the confirmation bias? Gestalt psychologists might suggest paying attention to and manipulating the figural aspects of a problem. This is in line with what more recent studies on insights in problem solving have suggested (Luo, Niki & Knoblich 2006; Öllinger, Jones & Knoblich 2008). These studies demonstrated the importance of representational change in overcoming impasse phases. In particular, they propose that the initial representation of a problem (derived from past experience) can be changed by means of two mechanisms: relaxing the constraints originating from past knowledge that limit the space within which a solution is sought (relaxation of constraints), and breaking up familiar perceptual patterns of chunks of objects or events to create new patterns of features or components (chunk decomposition). Constraints which derive from past experience and familiar perceptual chunks contribute towards the formation of the initial mental representation and might lead to an impasse. To get over this, a representational change is necessary. This is defined as a cognitive process, but it is recognized that some perceptual-visual processes are implied as suggested by Luo et al. (2006) and confirmed in various studies demonstrating the role of graphical representation in problem solving (Weller, Villejoubert & Vallee-Tourangeau 2011).

It is at this point that our proposal comes in. Following on from these previous works, we suggest that a manipulation of the phenomenal structure of a problem based on a systematic transformation of its features into their opposites can be an effective strategy. It is guided and constrained by the phenomenal structure of the problem and in addition satisfies the requirements of epistemic vigilance. But what do we mean by "epistemic vigilance"? In recent literature, it has been defined as a filter mechanism which acts on information received. It assists in the evaluation of the communicator and the assessment of the content of the message. Thus the risk of being misinformed by others is avoided (Mercier & Sperber 2011). We suggest that a similar mechanism might apply to the examination and checking of various possible solutions in a problem-solving situation and that the phenomenal structure of the problem acts as a filter during this process. This occurs not only because the gaps to be healed and the direction to follow are revealed (as suggested by Gestalt psychologists), but also because humans tend to rely on what is evident (Brem & Rips 2000) and so when faced with a problem to resolve they are inclined to trust any line of research that is grounded in the structure of the problem itself, even when this is in contrast with their beliefs. In other words, the strength of solutions based on the phenomenal structure of the problem is that they are easy to justify because they are based on what can be observed.

But in practice, how does the phenomenal structure of a problem act as a filter? According to Gestalt psychologists, this has essentially to do with the reorganization of the elements of which the problem is composed. For Wertheimer (1945), as already mentioned, this reorganization basically consists of unifying elements that appear to be separated and separating elements that appear to be unified (which are, by the way, two opposite operations…). We suggest that the problem-solving process is facilitated by working on the original properties of a configuration and turning them into their opposites. This works because very often the original configuration of a problem needs to be inverted with respect to some critical aspects in order for a solution to become clear.

The idea that opposites might help was initially suggested by Duncker (1945) who, while discussing two kinds of fixedness, stated that the alterations that need to be made during a process of productive thinking are, in some cases, a shift of function from the original to a contrary function. We agree with Duncker that contrariety is not to be thought of as a logical relationship but as a perceptual relationship.

> "As clarification of the concept of 'shift of function within a system' I call two different functions of the same object 'contrary'. This is a generalization of the logical concept: 'contrary terms'. For example, one calls long and short (or red and blue) contrary because in a pure form they are mutually exclusive. They belong to the same 'dimension', concern the object in the same respect, have the same structural locus, and consequently are in specific and active relation to one another. Long and red, on the other hand, are not contrary; between them there is, in a way, a 'dead interval'. Until now the concept 'contrary' has been defined in logic only with reference to abstract or ideal wholes (namely property dimensions). I apply it analogously in relation to 'real wholes', i.e. to particular and sometimes unique real structures in which different functions demand each other in different places, such as, e. g. hammer and anvil, father and son, radius and tangent. Such functions may be called 'really contrary'. Now, if one and the same object is to take on in succession 'really contrary' functions, we shall call this a shift of function within a system" (Duncker 1945, 100).

These two 'really contrary' functions fit in well with the meaning that we assign to the term *contrary* when we state that a relationship of contrariety links the structure of a problem and its solution (Branchini, Burro & Savardi 2009; Branchini, Burro, Bianchi & Savardi, submitted). This perceptual interpretation of contrariety has been experimentally developed in contemporary research in Psychology. Evidence has been provided to demonstrate: a) that contrariety is directly perceivable between events/objects or properties and that it possesses specific and peculiar features that make it different from other perceived relationships such as similarity and diversity (Bianchi & Savardi 2008); b) that contrariety is grounded in the perceptual experience of space (Bianchi, Burro, Torquati & Savardi 2013; Bianchi, Savardi & Kubovy 2011) and c) that binary contrasts are elementary pre-linguistic schemas (Casasola 2008). The primal nature of contrariety is compatible with the universality of antonyms in common language. On the special status of opposites in language, see for example Croft & Cruse 2004; Paradis & Willners 2011.

Before moving on, we would like to exemplify how contrariety works in a problem-solving process by using one of Dunker's examples, namely, a problem where the application of a theorem to a concrete situation is required (Duncker 1945). Pasch's theorem states that "given a triangle and, in the same plan, a straight line which passes through no apex of the triangle, it follows that if the straight line intersects one side of the triangle, then it intersects still another side of the triangle" (see the diagram on the left in Fig. 2).

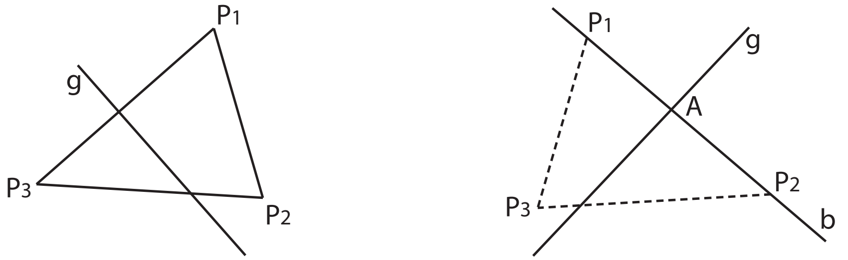

Fig. 2 The figure shows, on the left, a demonstration of Pasch's theorem and, on the right, the configuration to which it is to be applied (taken from Duncker 1945, 104).

The problem discussed by Duncker (see Fig.2, diagram on the right) comprises the following elements: 1) premises: given a straight line g, a point A along g, and a second straight line b, which is different from g, passing through A and two points P_1 and P_2 along b such that A lies between P_1 and P_2; 2) theorem: any arbitrarily chosen third point P_3 of the plane may be connected either with P_1 or with P_2 by a straight line in such a way that the connecting line does not intersect the straight line g.

According to Duncker, a model always implies a perceptual substratum. In this case there are two models: one represents Pasch's theorem and is called the P-model (Fig. 2, on the left); the other represents the specific configuration to which it is to be applied (Fig. 2, on the right). Following Duncker's analysis, the perceptual properties of the P-model and the second configuration are in contrast in the following sense:

1) the triangle P_1 P_2 P_3 is primary and absolute in the P-model, while it is secondary and relative in the other model;
2) the straight line g is secondary and relative in comparison to the triangle in the P-model, while it is primary and absolute in the second model;
3) the three sides of the triangle are homogeneous in the P-model (i.e. they have the same function), while they are non-homogeneous in the other model because the segment P_1P_2 appears to be more intimately bound up with g than with the other two sides; P_1 P_3 and P_2 P_3 function as connections of P_3 with P_1 and P_2, respectively, only when P_3 is established and positioned.

In order to reach the solution to the problem, it is necessary a) to take into account the properties of the problem and those of the theorem, b) to recognize that there is a relationship of contrariety between the two models for some of their features and c) to establish the apparent elements of contrast in the second model and transform them into the opposite. At this point it is clear that Pasch's theorem can also be applied to the second model.

Various other authors have intuitively (and sometimes also implicitly) suggested that problem-solving strategies are somehow based on contrariety (e.g. De Bono 1967; Kogan 1971; Zingales 1974). For instance, De Bono's (1967) approach to lateral thinking advises not stopping at the first potential solution but exploring and searching for alternative solutions by deliberately transforming the elements of the problem and *reversing* some of the relationships. Kogan (1971) suggested trying *opposition as means of reaching a solution* when direct means fail. Zingales (1974) proposed that a way of thinking creatively in problem solving is to stretch the situation or its individual aspects to their extreme values using strategies such as exaggeration, enlargement, or addition on the one hand and diminution, dissection and subtraction on the other (i.e. strategies which presuppose oppositional processes). These authors all intuitively touch upon how contrariety relates to problem-solving strategies. The same can be said for other scholars who have studied problem-solving as a case of hypothesis testing performance. Gale & Ball (2012) demonstrated that in Wason's 2- 4- 6 task (1960) in which participants are asked to discover the rule that governs the structure of number triples (the starting triple is: 2-4-6), success rates increase when a triple cue that contrasts with the target triple (i.e. 6- 4- 2) is provided.

The Plausibility of Looking for Contraries when Restructuring a Problem: Two Studies

Two empirical studies (Branchini, Burro & Savardi 2009; Branchini, Burro, Bianchi & Savardi, submitted) have indicated that stimulating participants in an experiment to use a systematic strategy involving the transformation of the properties present in the structure of a problem into their opposites positively affects the problem-solving process, reduces the time needed to reach a solution, changes the nature of potential solutions and affects the strategies activated during the solution process. These data, which we will briefly review and discuss, provide initial empirical support for the hypothesis discussed in this article.

Study 1: 30 undergraduate students were divided into 10 inter-observational groups each composed of three members. They were assigned a task in two different conditions: in the control condition 5 groups were presented with three problems and asked to solve them; in the experimental condition, the solution-seeking phase was preceded by a preliminary phase in which the other 5 groups were invited to identify all the contrary properties or contrary relationships present in or suggested by the structure of the problem. For instance, if the problem involved a figure with one part symmetrical and the other asymmetrical, or with some elements inside the border and others outside it, the participants were requested to identify these opposite characteristics (symmetrical-asymmetrical and inside-outside). They were also asked to pay attention to the cases where only one of the two properties was present (for instance if the figure was completely symmetrical) and to mentally recall what the opposite property was (i.e. asymmetrical). The task included traditional problems in problem-solving literature: Wertheimer's parallelogram (1945), Maier's nine dots (1930), and Harrower's duck (1932). They are reported in Appendix 1.

So why should an analysis of a problem in terms of its opposite properties help participants to find the solution? We may consider, for instance, the parallelogram problem (Fig. 3). To reach the solution, i.e. to calculate the area of the figure, it is necessary to transform the parallelogram (an *unstable*, *"leaning"* shape) into a rectangle (a *stable*, *"upright"* shape). To do this, one needs to draw two perpendicular lines from the A and B vertices and in this way two identical triangles are formed, one on the left *inside* the parallelogram and the other on the right *outside* the parallelogram. *By moving the left* hand triangle (which appears to be outside the rectangular shape and is thus "in excess") to "fill in" the triangular area *on the right* which appears to be "missing" from the rectangle, we obtain a regular rectangle, and the area can now be calculated in the normal way. These are not constraints which hold exclusively for this problem. As demonstrated in the review of various traditional problems published by Branchini et al. in 2009, if a critical contrariety is identified then the solution is revealed.

Fig. 3 Wertheimer's parallelogram problem: the area of the parallelogram can be found by transforming the parallelogram into a rectangle (as shown in the figure on the right; for further discussion, see text).

In the experiment described above, when participants were invited to analyze the structure of the problem in terms of contraries (the experimental condition), 4 out of the 5 groups found the correct solution to all three problems, while this happened for only 1 out of the 5 groups in the control condition. Moreover, participants spent less time solving the problems in the former than in the latter condition (Figure 4, top left diagram). These results suggest that the initial phase involving the identification of contrary properties is associated with a higher success rate in terms of both the number of correct solutions found and the time taken.

Differences were found also with respect to the types of solutions which emerged. The solutions proposed were classified as "reasoned solutions" (i.e. solutions resulting from an original reasoning process) versus "scholastic solutions" (i.e. solutions found by means of a mechanical application of previously learned rules), and as conventional solutions (i.e. standard solutions) versus alternative solutions (i.e. more creative solutions) and lastly as congruent solutions (compatible with the requirements of the problem) versus non-congruent solutions (violating the requirements of the problem). Significant differences also emerged in this case (Fig. 4, top right diagram): the groups in the experimental condition produced more reasoned than scholastic solutions and more alternative than conventional solutions while the opposite result emerged for the control group. In terms of which opposite properties participants focused on (Fig. 4, bottom diagrams), in the experimental condition participants more frequently took into consideration both spatial and non-spatial properties. Only one bias (not a significant difference) emerged for non-congruent solutions, which tended to be more frequent in the experimental condition. This latter result, if tested on a bigger sample, might confirm that in the process of "diverging" from (doing the contrary of) the initial structure of the problem, participants sometimes go too far and lose track of certain information that, instead, needs to be preserved.

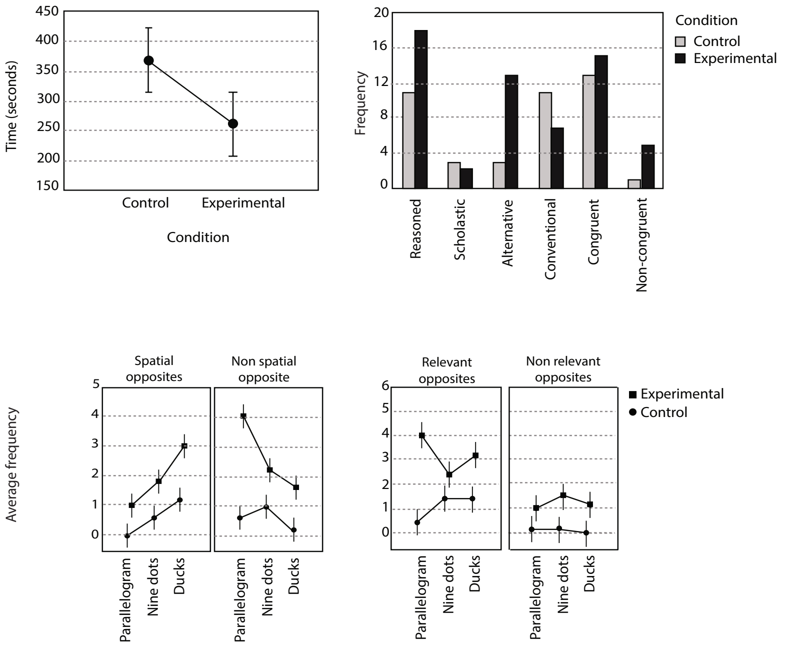

Fig. 4 Results of study 1 in terms of time needed to find the solution (top left diagram), type of solutions found (top right diagram) and the various different types of properties which were focused on and used to find a solution (diagrams at the bottom).

Study 2: Similar more promising results regarding the efficacy of the strategy based on contraries were obtained in the second study. 96 undergraduate students and 144 final year high school students took part in the experiment in inter-observational groups, with each group composed of three members. They were assigned the task in four different conditions. In two of these conditions, before starting their search for a solution, participants were asked to look for contraries (both explicit, i.e. the two contrary properties were present in the problem, and implicit, given the presence of only one property). In one of these two conditions, they were instructed to look for contraries and were advised that this strategy would help the solution process (AC). In the other condition, they were instructed to look for contraries but were not advised that this strategy would help the solution process (nAC). In the other two conditions, no reference to opposites was made. In one condition participants were instructed to use previous geometrical knowledge and were advised that this strategy would help the solution process (AK). In a fourth condition (n) participants were given no instructions regarding the use of a strategy and no advice that a particular strategy would help. In all four conditions, participants were invited at the beginning of the session to solve the task collectively by talking aloud and sharing their thoughts. They were allowed to use paper, pens, pencils and rulers during the session. There were no time limits. Each experimental session was video recorded.

The task included traditional problems in problem-solving literature: Wertheimer's parallelogram (1945), Maier's Nine Dots (Maier 1930), Harrower's Ducks (Harrower 1932), Wertheimer's Altar Window (Wertheimer 1945), Kanizsa's Square (Kanizsa 1973) and Köhler' s Circumference (Köhler 1969). They are reported in Appendix 1.

We studied the typology of the solutions that participants came up with. We distinguished five categories of response: correct perceptual solutions (cPS), i.e. correct solutions based on perceptual spatial operations without the application of previously learned rules; incorrect perceptual solutions (iPS), i.e. incorrect solutions based on perceptual spatial operations without the application of previously learned rules; correct prior knowledge solutions (cKS), i.e. correct solutions resulting from the application of previously learned knowledge or previously known rules; incorrect prior knowledge solutions (iKS), i.e. incorrect solutions based on the application of previously learned knowledge or previously known rules; and no solution (nS), referring to the cases where participants did not succeed in finding a solution (either correct or incorrect).

The analyses conducted (for more details see Branchini, Burro, Bianchi & Savardi, submitted) revealed that: a) when, during the solution process, participants were explicitly advised to look for contraries, the frequency of correct solutions was significantly higher than when they were not given any advice (AC > n) or in the not advised search of contraries (AC>nAC); b) perceptual solutions were more frequent when contraries were identified (both explicitly and implicitly), with no differences between the AC and nAC conditions, as compared to when participants were invited to use prior knowledge of formulas and rules (AC>AK: nAC>AK) or when no advice regarding strategies was given (AC>n; nAC>n); c) participants were faster when explicitly invited to look for contraries than when no suggestion was provided (solution latencies n>AC) or when they were invited to use previous geometrical knowledge (AK>AC); d) there were differences in terms of the procedures activated by participants during the solution process. In particular, prompting participants to look for contraries (either explicitly, AC, or implicitly, nAC) led them to concentrate more on what they were being asked to find. It also led them to expand on the problem by reformulating it rather than simply re-reading it and they also operated on visual aspects of the problem, e.g. modifying orientation and localization and separating and reorganizing parts of the structure or the whole structure. In contrast, there were fewer references to mathematical/geometrical formulas.

Conclusions

The aim of this article is to refocus attention on the role played by perceptual constraints and perceptual mechanisms in problem solving. In particular, developing an idea originally formulated by Gestalt psychologists that there is a strong link between the structure of a problem and its solution, we suggest that a careful analysis of the perceptual structure of a problem and an exploration of its counterfactual identity in terms of opposite properties and relationships a) gives clear indications with regard to possible operations to carry out in order to free reasoning from fixedness and impasse and reach a solution and b) satisfies the requirements of a mechanism of epistemic vigilance by keeping close to the factual nature which is evident in the problem. Indeed, the properties which are manipulated might not be critical ones (i.e. those which lead one to see the solution), but they are in any case related to the essence of the problem and integral to its structure. This type of exploration and manipulation is intuitive, easily applicable and does not require expert competence and it also conforms to the requisites of "epistemic vigilance" which filters, in a communicative setting, "reliable" reasoning processes from those which are unreliable.

Summary

Contemporary studies dealing with problem solving as a process of reasoning have focused on many cognitive aspects of this process but have disregarded the role of perceptual-figural aspects. Conversely, the importance of perceptual processes in problem solving has come to the foreground in studies on learning geometry (which have shown the importance of manipulating figural features in the solution process) and studies on

insight in problem solving (which have demonstrated the importance of restructuring a problem in order to solve it). This article aims to stimulate a reconsideration of the role of perception in problem solving by suggesting 1) that the degree of freedom established by the perceptual / representational structure of a problem guarantees a natural mechanism of epistemic vigilance in the cognitive reorganization process activated when a solution is sought and 2) that analyzing the perceptual structure of a problem by finding "the contrary" of its original properties helps people to restructure the problem and find a solution. The results of two studies are discussed in support of this second hypothesis.
Keywords: Problem solving, productive thinking, perception, contraries.

Zusammenfassung

Neuere Studien zum Problemlösen als Denkvorgang haben auf viele kognitive Aspekte dieses Prozesses fokussiert, haben die Rolle von Aspekten der figuralen Wahrnehmung aber unbeachtet gelassen. Andererseits haben Untersuchungen über das Lernen von Geometrie die Bedeutung des versuchsweisen Variierens von figuralen Merkmalen für den Lösungsvorgang gezeigt und so die Bedeutung von Wahrnehmungsvorgängen für das Problemlösen in den Vordergrund gerückt; Untersuchungen über die Rolle von Einsicht beim Problemlösen haben ebenfalls gezeigt, wie wichtig das Umstrukturieren eines Problems für dessen Lösung ist. Der vorliegende Beitrag möchte dazu anregen, die Rolle der Wahrnehmung beim Problemlösen neu zu überdenken, und bringt dazu folgende Überlegungen vor: 1) Die Wahrnehmungs- und Repräsentations-Struktur eines Problems beinhaltet einen bestimmten Freiheitsgrad, der bei der Suche nach einer Lösung einen natürlichen Mechanismus von Offenheit für Erkenntnisse im Zuge kognitiver Umstrukturierungen aktiviert. 2) Die Analyse der Wahrnehmungsstruktur eines Problems mit Hilfe des Findens "des Gegenteils" seiner ursprünglichen Eigenschaften hilft Menschen dabei, das Problem umzustrukturieren und eine Lösung zu finden. Die Ergebnisse von zwei Studien zur Unterstützung dieser zweiten Hypothese werden erörtert.
Schlüsselwörter: Problemlösung, produktives Denken, Wahrnehmung, Gegensätze.

References

Baccaglini-Frank, A., Mariotti, M.A. & Antonini, S. (2009): Different perceptions of invariants and generality of proof in dynamic geometry. In Tzekaki, M., Kaldrimidou, M. & Sakonidis, H. (Eds.), *Proceedings of the 33rd Conference of The International Group for the Psychology of Mathematics Education*, Vol. 2 (89-96). Thessaloniki, Greece: PME.

Bianchi, I. & Savardi, U. (2008): Contrariety as perceptual relationship. *Gestalt Theory*, 30 (2), 106-120.

Bianchi, I., Savardi, U. & Kubovy, M. (2011): Dimensions and Their Poles: a Metric and Topological Approach to Opposites. *Language and Cognitive Processes*, 26 (8), 1232-1265.

Bishop, A.J. (1989): A review of research on visualization in mathematics education. *Focus on Learning Problems in Mathematics*, 11 (1), 7-16.

Branchini, E., Burro, R., Bianchi, I. & Savardi, U. (submitted): Contraries as an effective strategy in geometrical problem solving. *Thinking & Reasoning.*

Branchini, E., Burro, R. & Savardi, U. (2009): Contraries in productive thinking. In Savardi, U. (Ed.), *The perception and cognition of contraries* (203-224). Milan, Italy: McGraw-Hill.

Brem, S.K. & Rips, L.J. (2000): Explanation and Evidence in Informal Argument. *Cognitive Science*, 24 (4), 573-604.

Burns, N.R., Nettelbeck, T. & McPherson, J. (2009): Attention and Intelligence. A Factor Analytic Study. *Journal of Individual Differences*, 30 (1), 44-57.

Casasola, M. (2008): The development of infants' spatial categories. *Current Directions in Psychological Science*, 17, 21-25.

Croft, W. & Cruse, D.A. (2004): *Cognitive linguistics*. Cambridge, UK: Cambridge University Press.

De Bono, E. (1967): *The use of lateral thinking*. London: Cape.

DeYoung, C.G., Braver, T.S., Shamosh, N.A., Green A.E. & Gray, J.R. (2009): Intellect as Distinct from Openness: Differences Revealed by fMRI of Working Memory. *Journal of Personality and Social Psychology*, 97 (5), 883-892.

Duncker, K. (1926): A qualitative (experimental and theoretical) study in productive thinking (solving of comprehensible problems). *Pedagogical Seminary*, 33, 642- 708.

Duncker, K. (1935): *Zur Psychologie des produktiven Denkes*. Berlin: Springer.

Duncker, K. (1945): On Problem-Solving. *Psychological Monographs*, 58 (5), 1-113.

Duval, R. (2006): A cognitive analysis of problems of comprehension in a learning of mathematics. *Educational Studies in Mathematics*, 61, 103-131.

Evans, J.St.B.T. (2008): Dual-processing accounts of reasoning, judgment and social cognition. *Annual Review of Psychology*, 59, 255–278.

Evans, J.St.B.T., Handley, S.J. & Bacon, A.M. (2009): Reasoning Under Time Pressure. A Study of Causal Conditional Inference. *Experimental Psychology*, 56 (2), 77-83.

Gale, M. & Ball, L.J. (2012): Contrast class cues and performance facilitation in a hypothesis-testing task: evidence for an iterative counterfactual model. *Memory and Cognition*, 40, 408- 419.

Gardner, H. (1983): *Frames of mind: The theory of multiple intelligence*. New York: Basic Books.

Gutiérrez, A. (1996): Children's ability for using different plane representations of space figures. In Batturo, A.R. (Ed.), *New directions in geometry education* (33-42). Brisbane, Australia: Centre for Math. and Sc. Education, Q. U. T.

Gutiérrez, A., Pegg, J. & Lawrie, C. (2004): Characterization of students' reasoning and proof abilities in 3-dimensional geometry. In *Proceedings of the 28th Conference of International Group for the Psychology of Mathematics Education,* Vol. 2 (511-518). Cape Town, South Africa.

Hahn, U. & Oaksford, M. (2007): The rationality of informal argumentation: A bayesian approach to reasoning fallacies. *Psychological Review*, 114 (3), 704-732.

Harris, L.J. (1981): Sex-related variations in spatial skill. In Liben, L.S., Patterson, A.H. & Newcombe, N. (Eds.), *Spatial representation and behavior across the life span* (83-125). New York: Academic Press.

Harrower, M.R. (1932): Organization in higher mental processes. *Psychologische Forschung*, 17, 56-120.

Kanizsa, G. (1973): Il problem solving nella psicologia della Gestalt [Problem solving in Gestalt psychology]. In Mosconi, G. & D'Urso, V. (Eds.), *La soluzione dei problemi* (35-86). Firenze, IT: Giunti Barbera.

Kogan, N. (1971): Educational implications in cognitive styles. In Lesser, G.S. (Ed), *Psychology and educational practice* (242-292). Glenview, Illinois: Scott & Foresman.

Köhler, W. (1920): *Intelligenzprüfungen an Menschenaffen*. Berlin: Springer.

Köhler, W. (1969): *The task of gestalt psychology*. Princeton: Princeton University Press.

Laughlin, P.R., Hatch, E.C., Silver, J.S, & Boh, L. (2006): Groups perform better than the best individuals on Letters- to- Numbers problems: Effects of Group size. *Journal of Personality and Social Psychology*, 90 (4), 644-651.

Luchins, A.S. (1942): Mechanization in problem solving. *Psychological Monographs*, 54 (248), 1-95.

Luchins, A.S. & Luchins, E.H. (1970): *Wertheimer's seminars revisited problem solving and thinking. Volume I.* New York: State University of New York At Albany, Inc.

Luo, J., Niki, K. & Knoblich, G. (2006): Perceptual contributions to problem solving: Chunk decomposition of Chinese characters. *Brain Research Bulletin*, 70, 430-443.

Maier, N.F.R. (1930): Reasoning in humans. On direction. *Journal of Comparative Psychology*, 115-143.

Mercier, H. & Sperber, D. (2011): Why do humans reason? Arguments for an argumentative theory. *Behavioral and Brian Sciences*, 34, 57-111.

Nerney, G. (1979): The Gestalt of problem solving: an interpretation of Max Wertheimer's productive thinking. *Journal of Phenomenological Psychology*, 10 (1), 56- 80.

Newell, A. & Simon, H.A. (1972): *Human Problem Solving*. Englewood Cliffs: Prentice Hall.

Öllinger, M., Jones, G. & Knoblich, G. (2008): Investigating the Effect of Mental Set On Insight Problem Solving. *Experimental Psychology*, 55 (4), 269-282.

Paradis, C. & Willners, C. (2011): Antonymy: From convention to meaning-making. *Review of Cognitive Linguistics*, 9 (2), 367- 391.

Unal, H., Jakubowski, E. & Corey, D. (2009): Differences in learning geometry among high and low spatial ability pre-service mathematics teachers. *International Journal of Mathematical Education in Science and Technology*, 40 (8), 997-1012.

Unsworth, N., Spillers, G.J. & Brewer, G.A. (2010): The Contributions of Primary and Secondary Memory To Working Memory Capacity: An Individual Differences Analysis of Immediate Free Recall. *Journal of Experimental Psychology: Learning, Memory and Cognition*, 36 (1), 240-247.

Wason, P.C. (1960): On the failure to eliminate hypotheses in a conceptual task. *Quarterly Journal of Experimental Psychology*, 12, 129–140.

Weller, A., Villejoubert, G. & Vallée-Tourangeau, F. (2011): Interactive insight problem solving. *Thinking & Reasoning*, 17 (4), 424-439.

Wertheimer, M. (1945): *Productive thinking*. New York and Evanston: Harper & Row Publishers. First appearance: Über Schlussprozesse im produktiven Denken, 1920

Zingales, M. (1974): *L'organizzazione della creatività*. Bologna, Italy: Cappelli.

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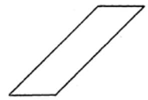
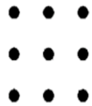
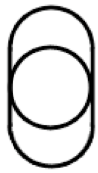
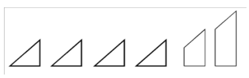
Appendix

Author	Year	Problem	Formulation	Figure
Wertheimer	1945	Parallelogram problem	In this problem we want to find the formula for calculating the area of the figure presented and to demonstrate why it is correct.	
Maier	1930	Nine dots problem	There are nine dots through which four lines must be drawn in such a manner that all dots will be passed through. The pencil must not be taken from the paper and no line should be retraced.	
Harrower	1932	Ducks problem	Swimming under a bridge there came two ducks in front of two ducks, two ducks behind two ducks, and two ducks in the middle. How many ducks were there in all?	No figure
Wertheimer	1945	Altar window problem	There are painters at work, painting and decorating the inner walls of a church. Above the altar there is a circular window. For decoration, the painters have been asked to draw two vertical lines tangent to the circle, and of the same height as the circular window; they were then to add half circles above and below, closing the figure. This area between the lines and the window is to be covered with gold. For every square inch, so much gold is needed. How much gold will be needed to cover this space (given the diameter of the circle); or, what is the area between the circle and the lines?	
Kanizsa	1973	Square problem	Build a square putting together six smaller figures: four right-angled isosceles triangles and two right-angled trapezoids having equal heights, but bases of different lengths.	
Köhler	1969	Circumference problem	There is a circle with a radius r, and we build a rectangle in this circle. If we trace the line l inside the rectangle, what is its length?